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Sincerely,
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Christophe Pierre and Steven W. Shaw

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The University of Michigan, Division of Research Development and Administration
3003 South State Street
Ann Arbor, MI 48109-1274

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13. ABSTRACT (Maximum 200 words)

The objective of this research was the development and implementation of computational modal analysis methodologies for large-scale, complex, nonlinear structures. These methods are based on nonlinear modes of vibration defined and constructed in terms of invariant manifolds. The motivation for the research stems from the fact that the dynamics of nonlinear structures are typically decomposed in terms of the linearized system s modes, often yielding poor modal convergence and too large reduced-order models. Research during the grant years focused on (1) the computational implementation of nonlinear modal analysis for large-scale systems, including those modeled via the finite element method, (2) the development of new nonlinear Galerkin techniques for large-amplitude response, and (3) the application of these methods to rotating rotorcraft blades.

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FINAL REPORT

Nonlinear Modal Analysis and Component Mode Synthesis of Large-Scale Structural Systems

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SCIENTIFIC PROGRESS AND ACCOMPLISHMENTS

The primary goal of this project has been to develop methods for generating reduced-order models that accurately describe the vibrations of large-scale nonlinear structural systems. The approach makes use of nonlinear normal modes as described in terms of invariant manifolds of the dynamic system model. In previous work we have successfully developed the basic theory and applied it to a number of examples. Our more recent efforts have concentrated in three overlapping themes: (1) formulations that are computationally (*i.e.*, FE) based, so that the technique can be applied to complex, large-scale models of engineering importance; (2) rotor blade applications, of interest to ARO; and (3) extensions of the techniques to large amplitudes, utilizing a novel Galerkin approach, which extends the method significantly in terms of range of validity in each specific problem, as well as in terms of application areas. A summary of accomplishments is given below.

1. Computationally Based Techniques

Computational implementation

We have automated the generation of the coefficients required for developing nonlinear multi-mode models that are based on series approximations of system nonlinearities. This advance allows one to systematically and rapidly generate reduced-order models from a large-scale set of dynamic equations. This approach has been applied to sample beam problems formulated both analytically and by the finite element method. This approach is particularly powerful when applied to systems with internal resonances, where interactions between modes are essential.

Finite element applications

The nonlinear modal analysis techniques have been developed keeping in mind that the ultimate outcome will be user-friendly computer codes that will allow for the efficient simulation of the nonlinear vibrations of complex structures. In that vein, we have successfully implemented and demonstrated the methods using finite element models for beams, plates, and rotor blades, for both discrete and distributed nonlinearities. This is crucial, since the majority of practical vibration problems are cast in finite element form.

It is also unique, since most analytical techniques developed for nonlinear vibrations are incompatible with finite element formulations.

2. Rotor Blade Vibrations

Improved convergence for reduced-order models

In the coupled transverse (flapping)-axial vibrations of a rotating blade, one needs to incorporate a large number of degrees of freedom in order to obtain a high-fidelity vibration model. Even in the simplest rotor-blade models, the inherent nonlinear coupling in these directions can cause significant convergence problems when one uses conventional finite element or linear modal expansion approaches, resulting in models with a large number of degrees of freedom. These large-scale models are undesirable in terms of the computational effort required for simulation studies. The use of nonlinear normal modes provides a systematic and rigorous approach that correctly accounts for this nonlinear coupling, and it captures the essential dynamics with a much smaller number of degrees of freedom. It has been shown, for example, that a single nonlinear mode, defined by invariant manifolds, can accurately describe the fundamental flapping mode of a rotating beam for quite large amplitudes, whereas one would need at least 18 linear modes to obtain the same level of accuracy using traditional techniques. These promising results suggest that this approach may be of significant benefit in the development of accurate, low-order dynamic models of rotorcraft blades.

Nonlinear modes for finite element rotor-blade models

The governing nonlinear equations of motion for the simple rotorcraft blade model were also discretized by the finite element method. Standard one-dimensional finite elements were used, with three coordinates at each node (axial displacement, bending slope, and bending displacement), yielding quadratic and cubic nonlinear terms in the nodal coordinates. The equations in finite element coordinates were then transformed into equations in the linear modal coordinates. These were truncated to eliminate unreliable higher-order information, and nonlinear modal analysis was applied to the resulting equations of motion, using the Galerkin technique. It was found that this approach converges to the exact result as the number of elements in increased. Again in this case, the ultimate model, after nonlinear modal reduction, has only a *single* degree of freedom, whose coefficients depend on the finite element treatment. These results demonstrate the applicability of nonlinear modal analysis to finite element structural representations, including those with distributed nonlinearities. It also demonstrates is usefulness to a system of direct importance to ARO.

3. Large-Amplitude Formulations for Nonlinear Modal Analysis

Nonlinear Galerkin Techniques

In this new approach, the invariant manifold equations are discretized via a Galerkin expansion using basis functions in amplitude and phase coordinates and subsequent projection, yielding a set of nonlinear algebraic equations for coefficients that are solved

using the Hybrid Powell method. This approach replaces the asymptotic series solution used in past work, and it avoids its inherent shortcomings. It greatly extends the applicability of the overall approach and opens doors to many new opportunities. Key features of this new method are: the vibration amplitude range is specified by the user, and the accuracy over that range can be controlled by the user as well, by simply varying the number of terms in the Galerkin expansion. This eliminates all guesswork concerning the accuracy and range of validity of the reduced-order model generated. This breakthrough significantly extends the reliability and applicability of invariant manifold-based nonlinear modal analysis, and it brings the ideas significantly closer to implementation as a standard tool for nonlinear structural vibration analysis. It has been tested on a number of problems, including simple academic mass-spring models, finite element beam models, and a rotor blade model, as described below.

Large amplitude rotor-blade modeling using nonlinear Galerkin solution

In the coupled transverse (flapping)-axial vibrations of a rotating blade, one needs to incorporate a large number of degrees of freedom in order to obtain a high-fidelity vibration model. Even in the simplest rotor blade models, the inherent nonlinear coupling in these directions can cause significant convergence problems when one uses conventional finite element or linear modal expansion approaches, resulting in models with a large number of degrees of freedom. These large-scale models are undesirable in terms of the computational effort required for simulation studies. A simple model for the nonlinear coupled axial/transverse vibrations of a uniform rotor blade was investigated. The two governing nonlinear, coupled, partial differential equations of motion were discretized by expansions using a relatively large set of assumed modes. The Galerkin approach was then used to develop a single-degree of freedom model for the first nonlinear flapping mode. Simulation results for the resulting model are indistinguishable from the exact solution up to a very large amplitude, at which the "peak-to-peak" blade tip amplitude is more than 10% of the blade length. In fact, the Galerkin approach significantly outperforms all other approaches to this problem, by using a *single* nonlinear mode. The large-amplitude reliability of this reduced-order model and the versatility of this new approach are the essential driving forces behind the current research program.

TECHNOLOGY TRANSFER

In October 1998 a one-day visit was made to the U.S. Army Aeroflightdynamics Directorate at NASA Ames Research Center. Results of the application of the nonlinear normal mode approach to the rotating beam problem were presented, and these were deemed encouraging. Further discussions were held with Army researchers Robert Ormiston and Michael Rutkowski. It was decided that this line of research should be pursued by applying the technique to more realistic blade models. Since that visit, Dr. Gene Ruzicka of NASA Ames, working in collaboration with Professor Dewey Hodges at Georgia Tech, has examined the relationship between the nonlinear normal modes approach and hybrid finite-element techniques being proposed to handle convergence problems in rotor blade models. They have found some promising results that have been

submitted as a conference paper for the upcoming ASME Biennial Conference on Noise and Vibration, Pittsburgh, September, 2001.

In October 1999 Professor Pierre presented the paper, "Nonlinear Modal-Based Reduction of Models for Rotating Blade Vibrations," at the *Eighth ARO Workshop on Aeroelasticity of Rotorcraft Systems*, held at the Pennsylvania State University, where our work was presented to a group of rotorcraft experts.

In December 1999 Professors Shaw and Pierre made a one-day visit to NASA/Army laboratories at NASA Langley Research Center in Hampton, VA. We presented a seminar that described the general nature of our research, its successful applications, and its potential for new applications. We discussed potential opportunities for technical exchanges and collaboration with NASA/Army personnel, and also toured the facilities. Very recently, Dr. Walt Silva of NASA Langley has contacted Professor Shaw about participation in a nonlinear modeling workshop to be held at Langley in May 2001.

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SCIENTIFIC PERSONNEL

Christophe Pierre, principal investigator, Professor, Department of Mechanical Engineering and Applied Mechanics, the University of Michigan.

Steven W. Shaw, co-principal investigator, Professor, Department of Mechanical Engineering, Michigan State University.

Eric Pesheck, Graduate Student Research Assistant, Department of Mechanical Engineering and Applied Mechanics, the University of Michigan, Ph.D., 1999.

Polarit Apiwattanalunggarn, Graduate Student Research Assistant and Doctoral Candidate, Department of Mechanical Engineering, Michigan State University.

Dongying Jiang, Graduate Student Research Assistant, Department of Mechanical Engineering and Applied Mechanics, the University of Michigan.

INVENTIONS

None.